1. What is the number of basic steps executed by the following method (as a function of n)? What is the time complexity of the method?

```java
public int howLongA(int n) {
    int k = 0, kk = 0;
    for(int i = 0; i < n/2 ; i++) {
        k++;
        for(int j = 0; j < n/2; j++)
            kk++;
    }
    return k*kk;
}
```

2. \( O(g(n)) = \{ f(n) : \) there exist positive constants \( c \) and \( n_0 \) such that \\
\[ 0 \leq f(n) \leq cg(n) \) for all \( n \geq n_0 \} \)

Use the definition of \( O \)-notation to determine which of the following equations are true. For each true statement, find the minimum values of \( c \) and \( n_0 \).

- \( a. \quad 2^{n+1} = O(2^n) \)
- \( b. \quad 2^{2n} = O(2^n) \)
- \( c. \quad n! = O(2^{n \log n}) \)

3. Insertion sort can be expressed as a recursive procedure as follows. In order to sort \( A[1...n] \), we recursively sort \( A[1...n-1] \) and then insert \( A[n] \) into the sorted array \( A[1...n-1] \). Write a recurrence for the running time of this recursive version of insertion sort.

4. Find asymptotic upper and lower bounds for recurrence relations a and b.

- \( a. \quad T(n) = 2T(n/7) + \sqrt{n} \)
- \( b. \quad T(n) = 9T(n/3) + n^2 \)
- Show that the Master Theorem does not apply to \( T(n) = 2T(n/2) + n \log n \).
- Use the substitution method to show that \( T(n) = 2T(n/2) + n \log n \) is \( O(n \log^2 n) \).

5. Solve the recurrence \( T(n) = T(\sqrt{n}) + 1 \).

6. Consider a max-heap \( A[1...n] \) on \( n \) numbers and assume all these numbers are distinct.

- a. Give all possible locations (possible I values) of the smallest numbers.
- b. Give all possible locations (possible I values) of the second smallest numbers.

7. You are given a max-heap \( A[1...n] \) on \( n \) elements and an integer \( 1 \leq k \leq n \). Give an \( O(k \log k) \) algorithm (in pseudocode that computes the k largest elements in \( A \) in sorted order.

8. Give an \( O(n \log k) \) algorithm to merge \( k \) sorted lists into one sorted list, where \( n \) is the total number of elements in all the input lists. Hint: Use a heap for k-way merging. Analyze the time complexity of your algorithm.
9. Let \( A[1\ldots n] \) be an array, where each entry \( A[i] \) is either red, white or blue.
   a. Given an \( O(n) \) algorithm (in pseudo code) that rearranges the entries in \( A \) such that all red entries are to the left of all white entries, and all white entries are to the left of all blue entries.
   b. What is the invariant maintained by the algorithm?
   c. Explain why the running time is \( O(n) \).

10. Give an algorithm to sort \( n \) numbers in the range 0 to \( n^3 - 1 \) in linear time.

11. What property must the input numbers satisfy for Bucket Sort to run in linear time?

12. Suppose we are given a hash table and we wish to insert the keys 3, 13, 31, 21, and 15 when the hash function is \( \text{fn } x \Rightarrow x \mod 5 \).
   a. Draw a picture of the hash table that results if we use linear probing to resolve conflicts.
   b. Draw a picture of the hash table that results if we use double hashing to resolve conflicts with a second has function \( \text{fn } x \Rightarrow (x+1) \mod 5 \).